DENSITY AND CHARGE FLUCTUATIONS IN MULTIPLE PRODUCTION

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Abstract

Here, I summarize briefly main results of the studies of density and charge fluctuations in multiparticle production in which I was mainly involved during last 3 years.

The solution of QCD equations for generating functions of *parton* multiplicity distributions reveals new features of cumulant moments oscillating as functions of their rank. Experimental data on *hadron* multiplicity distributions in e^+e^-, hh, hA, AA collisions possess the similar features. Contrary, the "more regular" models like $\lambda\phi_6^3$ predict a different pattern. Evolution of the moments at smaller phase space bins, zeros of the truncated generating functions and the singularity of the generating function are briefly discussed. Such studies can provide some guide to a new representation of multiparticle processes as compared to the common Fock representation.

Also, the large charge fluctuations can be observed if in some events pions tend to be produced in coherent or squeezed isospin states.

1 Introduction and main results

During the last decade we worked actively on correlations and fluctuations of the number of particles produced within small phase space bins inspired by the ideas of intermittency and fractality. The factorial and cumulant moments were especially helpful in this respect. Here I would like to show that the correlations and fluctuations in large reveal new unexpected features in the behavior of the moments as functions of their rank. The larger number of particles in the total phase space allows to analyze the moments of higher ranks compared to small bins. This is somewhat different view of the density fluctuations within the available phase space volume.

Another interesting problem is related to the charge fluctuations in a single event. It is closely connected to the so-called Centauro events observed in cosmic rays.

I present from the very beginning both short review of the problem and the main results obtained during the last 3 years, leaving their derivation and brief discussion for the next section. Those interested in more detailed description (and, in particular, in formulae and Figures demonstrating the statements and shown at oral presentation) should use the list of references

for further reading (e.g., the review papers [1,2,3]). The density fluctuations are given by the width (and by higher moments) of the multiplicity distribution, while charge fluctuations show e.g. how the share of neutral pions in single events declines from its standard value of 1/3. The first problem is treated in the framework of QCD, the second one is considered for coherent and squeezed isospin states.

I would like to stress that QCD predicts the distributions of partons (quarks and gluons) while in experiment one gets the distributions of final hadrons. Therefore no quantitative comparison has been attempted. To do that, one must rely on the Monte-Carlo models with some definite hadronization schemes. However, the qualitative features of both distributions are so spectacular and remind each other that one is tempted to confirm once again that QCD is a powerful tool for predicting new features of hadron distributions as well.

For a long time, the phenomenological approach dominated in description of multiplicity distributions in multiparticle production [4]. The very first attempts to apply QCD formalism to the problem failed because in the simplest double-logarithmic approximation it predicts an extremely wide shape of the distribution in the global phase space [5] (i.e. huge fluctuations) that contradicts to experimental data. Only recently it became possible [6] to get exact solutions of QCD equations for the generating functions of multiplicity distributions which revealed much narrower shapes and such a novel feature of cumulant moments as their oscillations at higher ranks [7,8]. The similar oscillations have been found in experiment for the moments of hadron distributions [9]. Their pattern differs drastically from those of the popular phenomenological distributions [1] and of the "non-singular" (however, possessing the asymptotic freedom property) $\lambda\phi_6^3$-model [10]. The QCD inspired Monte-Carlo models describe experiment quite well [11].

These findings have several important implications [1]. They show that:

1.*the QCD distribution belongs to the class of non-infinitely-divisible ones.*

Two corollaries of this statement follow immediately:

a. The Poissonian cluster models (e.g., the multiperipheral cluster model) are ruled out by QCD.

b. The negative binomial distribution (so popular nowadays in phenomenological fits) is not valid.

2.*the new expansion parameter appears in description of multiparticle processes.*

Since this parameter becomes large when large number of particles are involved, it asks for the search of some collective effects and of more convenient basis than the common particle number (Fock) representation.

QCD is also successful in qualitative description of evolution of multiplicity distributions with decreasing phase space bins which gives rise to notions of intermittency and fractality [12,13,14]. The fluctuations increase in smaller bins. However, there are some new problems with locations of the minimum of cumulants at small bins [15,16].

The experimentally defined truncated generating functions possess an intriguing pattern of zeros in the complex plane of an auxiliary variable [16,17,18]. It recalls the pattern of Lee-Yang zeros of the grand canonical partition function in the complex fugacity plane related to phase transition [19,20] and asks for some collective effects to be searched for [21,2]. At high multiplicities these zeros tend to pinch the positive real axis at the singularity position of the generating function and their study can reveal the nature of the singularity (see [21,22]) that has far-reaching consequences for the theory of multiple production.

Besides density fluctuations, there could appear the asymmetry between neutral and charged pions distributions. The ideas of chiral models, disoriented chiral condensate, coherent and squeezed isospin states could be useful in approaching the problem of charge fluctuations. I describe them briefly in a separate section.

2 Some QCD technicalities and results

Let us define the multiplicity distribution

$$P_n = \sigma_n / \sum_{n=0}^{\infty} \sigma_n, \qquad (1)$$

where σ_n is the cross section of n-particle production processes, and the generating function

$$G(z) = \sum_{n=0}^{\infty} P_n (1+z)^n. \qquad (2)$$

The (normalized) factorial and cumulant moments of the P_n distribution are

$$F_q = \frac{\sum_n P_n n(n-1)...(n-q+1)}{(\sum_n P_n n)^q} = \frac{1}{\langle n \rangle^q} \frac{d^q G(z)}{dz^q}\big|_{z=0}, \qquad (3)$$

$$K_q = \frac{1}{\langle n \rangle^q} \frac{d^q \ln G(z)}{dz^q}\big|_{z=0}, \qquad (4)$$

where $\langle n \rangle = \sum_n P_n n$ is the average multiplicity. They describe full and genuine q-particle correlations, correspondingly.

Here, I consider QCD without quarks i.e. gluodynamics, since quarks do not change qualitative conclusions described below. The generating function of the gluon multiplicity distribution in the global phase-space volume satisfies the equation

$$\frac{\partial G(z,Y)}{\partial Y} = \int_0^1 dx K(x)\gamma_0^2[G(z,Y+\ln x)G(z,Y+\ln(1-x)) - G(z,Y)]. \quad (5)$$

Here $Y = \ln(p\theta/Q_0), p$ is the initial momentum, θ is the angular width of the gluon jet considered, $p\theta \equiv Q$ where Q is the jet virtuality, Q_0 =const,

$$\gamma_0^2 = \frac{6\alpha_S(Q)}{\pi}, \quad (6)$$

α_S is the running coupling constant, and the kernel of the equation is

$$K(x) = \frac{1}{x} - (1-x)[2 - x(1-x)]. \quad (7)$$

The eq.(5) can be solved exactly for fixed coupling constant[8] and in higher order approximations for the running coupling[6]. In the last case, the solution of this equation in terms of moments looks as

$$H_q = \frac{K_q}{F_q} = \frac{\gamma_0^2[1 - 2h_1\gamma + h_2(q^2\gamma^2 + q\gamma')]}{q^2\gamma^2 + q\gamma'}, \quad (8)$$

where the anomalous dimension $\gamma \approx \gamma_0 + O(\gamma_0^2)$. The main prediction is the minimum of $H_q = K_q/F_q$ at

$$q_{min} = \frac{1}{h_1\gamma_0} + \frac{1}{2} + O(\gamma_0) \approx 5 \quad (9)$$

and subsequent oscillations of the ratio H_q at higher q. Let us note that it owes to the singular part of the kernel and is absent[10] in more regular theories like $\lambda\phi_6^3$.

While the above results are valid for gluon distributions in gluon jets (and pertain to QCD with quarks taken into account[1]), the similar qualitative features characterize the multiplicity distributions of hadrons in high energy reactions initiated by various particles and nuclei. The numerous demonstration of it can be found in papers[9,11].

Another important feature of the theoretical results is the presence of the product $\gamma_0 q$ as a new expansion parameter in all the solutions. Formally, it vanishes in asymptotics. However, since $\gamma_0 \approx 0.48$ even at Z^0-peak this

parameter is large at any rank q and determines the main properties of the moments. One can say that the asymptotics is unreachable, in practice. Let us remind that this asymptotics is somewhat similar to the negative binomial distribution with very wide shape compared to experimental ones (it gives rise to $H_q = q^{-2}$ while NBD with $k = 2$ predicts $H_q = 2/q(q+1)$). Since at higher ranks one deals with high multiplicity events the product $\gamma_0 q$ indicates that for such processes the usual Fock representation is not convenient. Therefore one is tempted to look for a more suitable representation for multiparticle processes.

The multiplicity distributions can be measured not only in the total phase space (as has been discussed above for very large phase-space volumes) but in any part of it. The most interesting problem here is the law governing the growth of fluctuations and its possible departure from a purely statistical behavior related to the decrease of the average multiplicity in small bins. Such a variation has to be connected with the dynamics of the interactions. In particular, it has been proposed [23] to look for the power-law behavior of the factorial moments for small rapidity intervals δy

$$F_q \propto (\delta y)^{-\phi(q)} \quad (\phi(q) > 0) \quad (\delta y \to 0), \tag{10}$$

inspired by the idea of intermittency in turbulence. In the case of statistical fluctuations with purely Poisson behavior, the intermittency indices $\phi(q)$ are identically equal to zero.

Experimental data on various processes in a wide energy range support this idea, and QCD provides a good basis for its explanation as a result of parton showers [12,13,14].

Let us turn now to the q-behavior of moments at small bins. The phenomenon of the oscillations of cumulants discussed above reveals itself here as well. According to the theory [6,15], the first minimum moves to higher ranks at higher energies because more massive jets become available. Another corollary is that it should shift to smaller values of q for smaller bins at fixed energy because the effective value of the anomalous dimension increases due to lower effective masses of subjets (since $q_{min} \propto \gamma_0^{-1}$). While former statement finds some support in experiment, the second one does not look to be true (probably, due to higher order terms in the relation above). Also, one should always keep in mind that lower multiplicities in smaller bins prevent from getting higher order moments with good enough precision.

There is another fascinating feature of multiplicity distributions – it happens that zeros of the truncated (if the sum in 2 runs up to $n = N_{max}$) generating function form a spectacular pattern in the complex plane of the variable z. Namely, they seem to lie close to a single circle. At enlarged values

of N_{max} they move closer to the real axis pinching it at some positive value of z.

No QCD interpretation of the fact exists because it is hard to exploit the finite cut-off in analytic calculations. The interest to it stems from the analogy to the locations of zeros of the grand canonical partition function as described by Lee and Yang who related them to possible phase transitions in statistical mechanics. In that case, the variable z plays the role of fugacity, and pinching of the real axis implies existence of two phases in the system considered. The Feynman-Wilson liquid analogy can be used when applying this idea to particle production. However, in my opinion, it would be premature to consider this phenomenon as a signature of any phase transition in particle collisions.

In particle physics, it shows up the location of the singularity of the generating function. The number of zeros of truncated generating functions increases and they tend to move to the singularity point when $N_{max} \to \infty$. Since it happens to lie close to the origin, it drastically influences the behavior of moments (see (3), (4)), and, therefore, determines the shape of the distribution. It explains also why the moment analysis is so sensitive to the tiny details of this shape. At the same time the stability of the qualitative features of the moments for different reactions is very impressive and implies some common dynamics. The study of the singularities is at the very early stage now (new results have been reported at this Workshop in [22]), and one can only say that the singularity is positioned closer to the origin in nucleus-nucleus collisions and it is farthest in e^+e^- that appeals to our intuitive guess.

Let us discuss possible implications of the results obtained. The very existence of the new expansion parameter shows that the particle number representation traditionally used in particle physics becomes inadequate for multiparticle production. It asks for the search of another approach. The appealing example of the analogous situation has been provided by laser physics where the coherent state representation is successful. However, in multiple production neither coherent nor squeezed states look quite promising since they do not fit above findings about distributions. Probably, their weighted averages giving rise to somewhat similar to the negative binomial distribution would be suitable. Anyway, the singularity of the generating function can become a starting point for further progress in that respect. This is an attractive direction for new research.

To conclude, I would like to stress that, once again, QCD demonstrates its power in predicting new features of *particle* distributions when dealing with *parton* distributions.

3   Notes on charge fluctuations

Above, we have discussed the charged multiplicity distributions which determine the density fluctuations of charged pions. We assumed implicitly that the same is true for neutral pions as well. However, if confirmed, Centauro events in cosmic rays imply that the strong charge asymmetry can be observed in some (probably, rather rare) events at very high energies. The perturbative QCD is unable to solve the problem since charge asymmetry appears at the hadronization stage only. There is no strong charge asymmetry in the common models. Therefore, one should rely on different dynamics. The analogy to photon states can be useful.

It is well known that the soft photons are produced in a coherent state. If the soft pions tend to be produced in a coherent (or squeezed due to the non-linear interaction) state and respect the isospin conservation, then one can show[24,25] (see also[3]) that the strong charge asymmetry should be seen in the individual events with a noticeable fraction of them having no charged pions at all. This is done by considering the projections of these states on the states with the definite value of the isospin. Due to isospin conservation, the total isospin of the pion system created at high energy is strongly restricted and should be much less than the number of pions. The differential distribution of the ratio f of the number of neutral pions to the total number of pions decreases very slowly as $f^{-1/2}$. The similar situation is typical for the chiral model used for describing the disoriented chiral condensate and the "Baked Alaska" scenaria[26,27]. These approaches favor the creation of squeezed isospin states as well[28]. However, neither coherent nor squeezed states are able to reproduce the pattern of moments oscillations described above. No more room is available here to discuss the problem. For more details about coherent isospin states, I refer to the paper[29] presented at this Workshop.

I should apologize that in these notes I described briefly the main statements about density and charge fluctuations in particle production leaving aside numerous details contained in the papers presented in the list of references. However, the size of the presentation prevents the detailed review.

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